PASCAL'S WAGER

Blaise Pascal was a French mathematician and scientist who lived from 1623 to 1662. His work involved concepts encountered in many high school and college classes: his first major work was a 1640 essay on conic sections, something many students are introduced to in precalculus classes; his name is attached to a hydraulic principle (Pascal's Principle) that many students learn in physical science classes. And he is known for "Pascal's wager", presented here.



Pascal's wager is found in his *Pensées*. According to T.S. Eliot, who wrote an introduction to an English translation of the *Pensées*, Pascal laid out the *Pensées* around 1660, just two years before he died, and so we must regard *Pensées* as "merely the first notes for a work which he left far from completion... a tower of which the stones have been laid on each other, but not cemented, and the structure unfinished." Eliot states that "the completed book was to have been a carefully constructed defence of Christianity... setting forth the reasons which will convince the intellect." But Pascal himself writes of tightly-reasoned proofs as not being what leads people to God:

The metaphysical proofs of God are so remote from the reasoning of men, and so complicated, that they make little impression; and if they should be of service to some, it would be only during the moment that they see such demonstration; but an hour afterwards they fear they have been mistaken.... This is the result of the knowledge of God obtained without Jesus Christ; it is communion without a mediator with the God whom they have known without a mediator. Whereas those who have known God by a mediator know their own wretchedness.... We know God only by Jesus Christ. Without this mediator all communion with God is taken away; through Jesus Christ we know God. All those who have claimed to know God, and to prove Him without Jesus Christ, have had only weak proofs.... [from Pensées numbers 542, 546]

While Pascal may have intended the wager as part of a defense of Christianity, the wager is now also viewed as an important early work in decision theory.¹ Pascal uses the idea of a wager only to lead an intelligent non-believer, who has no inclination toward belief, to the point of seeing that belief is rational. Once that point is reached, Pascal encourages this non-believer to not pursue further rational proofs of God's existence, but to pursue true belief through religious practice.

Pascal's "wager" discussion (written as a dialogue), from *Pensées* number 233, is in the left-hand column below.² The right-hand column contains a brief synopsis to aid student readers.

"God is, or God is not." But to which side shall we incline? Reason can decide nothing here. There is an infinite chaos separating us. A game is being played at the extremity of this infinite distance where heads or tails will turn up. What will you wager? According to reason, you can do neither the one thing nor the other; according to reason, you can defend neither of the propositions.

Do not then reprove for error those who have made a choice; for you know nothing about it. Pascal begins by stating that either God exists, or God does not exist. He asks you whether you should lean toward God existing, or lean toward God not existing. Think of it as a coin toss game, he says, where you have no particular reason to choose heads or tails. Moreover, the coin will be tossed, regardless of whether you want it to be tossed or not. The only question is, which side do you want to bet on?

No, but I blame them for having made, not this choice, but a choice; for again both the one who chooses heads and the one who chooses tails are equally at fault, they are both in the wrong. The true course is not to wager at all.	You answer that the proper thing to do is not to bet at all!
Yes; but you must wager. It is not optional. You are embarked. Which will you choose then? Let us see. Since you must choose, let us see which interests you least. You have two things to lose, the true and the good; and two things to stake, your reason and your will, your knowledge and your happiness; and your nature has two things to shun, error and misery. Your reason is no more shocked in choosing one rather than the other, since you must of necessity choose. This is one point settled. But your happiness?	Pascal says you have no choice. You are in fact on life's journey, and there either is a God or there is not a God. So do you bet on God existing, or do you bet on God not existing? Pascal says that you should consider the question of your happiness.
Let us weigh the gain and the loss in wagering that God is. Let us estimate these two chances. If you gain, you gain all; if you lose, you lose nothing. Wager, then, without hesitation that He is. <i>That is very fine. Yes, I must wager; but</i>	Consider what happens if you bet that God exists, says Pascal. If you bet that God exists, and God does in fact exist, you win everything. And if God does not exist, you lose nothing. So, Pascal says, bet on God existing. You say that this all sounds good, but what if you bet too much?
I may perhaps wager too much. Let us see. Since there is an equal risk of gain and of loss, if you had only to gain two lives, instead of one, you might still wager. But if there were three lives to gain, you would have to play (since you are under the necessity of playing), and you would be imprudent, when you are forced to play, not to chance your life to gain three at a game where there is an equal risk of loss and gain.	Pascal says that, since you have an equal chance of winning or losing, you could reasonably bet your life on this, if winning meant that you would win another life. In that case, if you won, you would win another life (and have two lives), and if you lost, you would lose your life (and have none). What you might gain, and what you might lose, are equal. But, he says, what if winning meant that you won two more lives (so you would have three)? Now, he says, you would be foolish not to bet your life, because you stand to gain more than you stand to lose. And remember, you do not have the option of not playing the game.
But there is an eternity of life and happiness. And this being so, if there were an infinity of chances, of which one only would be for you, you would still be right in wagering one to win two, and you would act stupidly, being obliged to play, by refusing to stake one life against three at a game in which out of an infinity of chances	But what you have to gain if God exists, says Pascal, is not just another life, but in fact an eternity of life and happiness. And since this is the case, it would be reasonable to bet on God's existence even if the odds were wildly against you. You are betting a finite thing (your life) for a chance at an infinite thing (eternal life). To not bet your life is to renounce reason. If God does not exist, your life is nothingness anyway. So if you give your life to God, and God does not exist, you have lost nothing.

there is one for you, if there were an infinity of an infinitely happy life to gain. But there is here an infinity of an infinitely happy life to gain, a chance of gain against a finite number of chances of loss, and what you stake is finite. It is all divided; wherever the infinite is and there is not an infinity of chances of loss against that of gain, there is no time to hesitate, you must give all. And thus, when one is forced to play, he must renounce reason to preserve his life, rather than risk it for infinite gain, as likely to happen as the loss of nothingness.

For it is no use to say it is uncertain if we will gain, and it is certain that we risk, and that the infinite distance between the certainty of what is staked and the uncertainty of what will be gained, equals the finite good which is certainly staked against the uncertain infinite. It is not so, as every player stakes a certainty to gain an uncertainty, and yet he stakes a finite certainty to gain a finite uncertainty, without transgressing against reason. There is not an infinite distance between the certainty staked and the uncertainty of the gain; that is untrue. In truth, there is an infinity between the certainty of gain and the certainty of loss. But the uncertainty of the gain is proportioned to the certainty of the stake according to the proportion of the chances of gain and loss. Hence it comes that, if there are as many risks on one side as on the other, the course is to play even; and then the certainty of the stake is equal to the uncertainty of the gain, so far is it from fact that there is an infinite distance between them. And so our proposition is of infinite force, when there is the finite to stake in a game where there are equal risks of gain and of loss, and the infinite to gain. This is demonstrable; and if men are capable of any truths, this is one.

Pascal notes that you might respond by saying that it is not certain that you will win, but it is certain that you will bet your life. That is, if you give your life over to God, it is certain that you gave your life, but it is uncertain that God actually exists. Does that gap between the certainty and the uncertainty not balance out the chance for infinite gain, so that it is reasonable not to bet on God's existence?

No, says Pascal. Every gambler who bets on any game knows that the betting, and the risk involved with betting, is certain, whereas the winning is uncertain. Yet that gambler can reasonably make bets on an ordinary game in which the winnings are limited. So if the gap between the certainty of the betting and the uncertainty of the winning does not make an ordinary bet unreasonable, it surely does not make this bet, with its chance for unlimited gain, unreasonable.

Pascal says that if people can see the truth in anything, they can see the truth in his argument.

I confess it, I admit it. But, still, is there no means of seeing the faces of the cards?

Yes, Scripture and the rest, etc.

Yes, but I have my hands tied and my mouth closed; I am forced to wager, and am not free. I am not released, and am so made that I cannot believe. What, then, would you have me do?

True. But at least learn your inability to believe, since reason brings you to this, and yet you cannot believe. Endeavour then to convince yourself, not by increase of proofs of God, but by the abatement of your passions. You would like to attain faith, and do not know the way; you would like to cure yourself of unbelief, and ask the remedy for it. Learn of those who have been bound like you, and who now stake all their possessions. These are people who know the way which you would follow, and who are cured of an ill of which you would be cured. Follow the way by which they began; by acting as if they believed, taking the holy water, having masses said, etc. Even this will naturally make you believe, and deaden your acuteness.

But this is what I am afraid of.

And why? What have you to lose?

But to show you that this leads you there, it is this which will lessen the passions, which are your stumbling-blocks.

Now, what harm will befall you in taking this side? You will be faithful, honest, humble, grateful, generous, a sincere friend, truthful. Certainly you will not have those poisonous pleasures, glory and luxury; but will you not have others? I will tell you that you will thereby gain in this life, and that, at each step you take on this road, you will see so great certainty of gain, so much nothingness in what you risk, that you will at last recognize that you have You answer that what Pascal says is compelling. But you are stuck. You are forced to play this game with your life. But you are not drawn to Scripture and to faith. You are by nature a non-believer. You ask Pascal what you should do.

Pascal says that your reason shows you that you *should* believe, and yet you *cannot* believe. So clearly the source of your unbelief lies outside of reason; it lies in your irrational side—in your "passions".

Therefore, he says, do not seek out additional reasoned proofs of God's existence and the like. Rather, do what others have done. Behave *as though* you believed. Go to church. Practice the religion. You will discipline your passions, and will come to believe naturally.

You answer that this is what you fear—coming to believe just out of habit, without much thought.

Pascal's reply is, why do you fear this? After all, what do you have to lose?

wagered for something certain and infinite, for which you have given nothing.

Ah! This discourse carries me away, charms me, etc.

If this discourse pleases you and seems impressive, know that it is made by a man who has knelt, both before and after it, in prayer to that Being, infinite and without parts, before whom he lays all he has, for you also to lay before Him all you have for your own good and for His glory, that thereby strength may be given to lowliness.

¹ See Itzhak Gilboa, *Theory of Decision under Uncertainty* (Cambridge University Press, 2009), section 5.2; and Donald Stabile, *Forerunners of Modern Financial Economics: A Random Walk in the History of Economic Thought* (Edward Elgar Publishing, 2005), chapter 2.

² The translation here is based primarily on the English translation (with introduction by T. S. Eliot) published in 1958 by E. P. Dutton & Co., with minor adjustments for readability made in consultation with the translation published in 1995 by Oxford University Press.